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# Optimizing Profit with the Linear Programming Model:
## A Focus on Golden Plastic Industry Limited, Enugu, Nigeria

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## ABSTRACT

*Linear programming, an operations research technique is widely used in finding solutions to complex managerial decision problems, but firms at Emene Industrial Layout make more use of the trial-and-error method. As such, firms at the Layout have been finding it difficult in allocating scarce resources in a manner that will ensure profit maximization or cost minimization. This study was carried out to seek and arrive at the optimal product-mix of a productive firm-the Golden Plastic Industry Limited- in the layout. The production problem of the firm was formulated as a linear programming problem and estimated as such. The result shows that only two sizes of the total eight "PVC" pipes should be produced. The study succeeded in establishing that Golden Plastic Industry Limited, Emene should produce 114,317.2 pieces of 25mm by 5.4m conduit pipes and 7,136.564 pieces of 20mm by 5.4m thick pressure pipes, and zero quantities of the rest sizes of pressure pipes per month in order to obtain a maximum profit of N1,964,537 given the present level of available funds and the technical coefficients of the products. The study also shows that only two of the raw materials –tio 2 and labour time- were surplus, while the other six-resin, calcium carbonate, stabilizer, cast, carbon black and blend-were scarce in relation to the formulated model. The shadow prices of the raw materials obtained showed their unit contribution to the objective function (profits) and suggests to management the prices at which they should either be bought or sold.*

**Keywords:** *Optimizing profit, Linear programming, Golden Plastic Industry*

## INTRODUCTION

Industries all over the world including Nigeria are continuously faced with shortages of production inputs which result in low capacity utilization and consequently low outputs. But an economy can only grow if management decisions at the firm level result in boosted output through either cost minimization or output maximization culminating in increased production in the real sector. Thus firm managers are always seeking for the right decisions so as to meet their objectives which mainly revolve on how best to increase profit.

The growth in industries puts pressure on management in finding the optimal planning, organizing, leading and controlling levels of production in the various productive industries of the economy. As a result of this pressure, managerial theories of the firm are introduced to analyze business environments and to solve practical business problems such as operational problems emanating from within the industry, and environmental problems in which the industry operates. Both theoretical and quantitative techniques[1] are developed to model and analyze these decision problems. Among these quantitative techniques is the linear programming model, which uses mathematical method in seeking the optimum course of action in any decision situation under the restriction of limited resources and uncertainties. Other mathematical approaches include; least squares model, time series, network system, inventory control, markov analysis and a host of others.

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[1] Theoretical techniques use verbal expressions while quantitative approaches make more use of mathematical symbols in arriving at solutions to practical business problems.

This study intends to investigate Golden Plastic Industry Limited **(**GPIL**)**, in particular and the manufacturing industries in Emene Industrial Layout[2] in general to assay the possibility of adopting linear programming and other mathematical approaches to profit optimization. Emene Industrial Layout was mapped out in the mid 1960's by the then Premier of Eastern Region in order to facilitate the ease of land acquisition for industrial purposes and, therefore, houses most of the industries in Enugu State, Nigeria. It is one of the most important industrial sites in Nigeria because of its strategic location[3] with a cluster of industries of the ever enterprising Igbo nation. The site has attracted many foreign industrial partnership with an international airport located at the very centre of the layout. The choice of linear programming is informed by the ability of the technique to solve problems relating to allocation of scarce resources to different types of products, and it is in a better position to save cost and streamline the product mix. From the profitability profile of Golden Plastic Industry, profit continued to vary in 2009; total profit in January was N1,200,000 but dropped by 10% in March to N1,080,000 and increased slightly again by 8% in April to N1,166.400. By May 2009, the total profit was N982,656 representing a decrease of over 15%. This instability problem has put pressure on management in finding the optimal production level that effectively utilizes the available resources to make maximum profit.

One decision area for the management of any firm is in determining the product mix of the firm. A product mix (also called product assortment), according to Kortler (1993) is the set of all product lines and items that a particular seller or producer offers for sale to buyers. Like most other developing countries of the world, Nigeria is faced with the general nature of scarcity of factors of production. As a result of the increasing cost of production, high customers' expectations and the general nature of the scarcity of factors of production, the management of industries can hardly do without a well-articulated course of action in their production decisions. This enables them to achieve their profit-maximizing objective.

The pronouncements of policy makers in Nigeria indicate that the aims of government economic policies are to increase the rate of growth of the economy and to increase the contribution (absolute and relative) of the industrial sector to GDP thus diversifying the economy. The contribution of the real sector to the rate of growth in the economy is very minimal. This is due to the sectoral dependence on crude-oil economy. The productive sector is neglected without a planned business environment to encourage national output, thus propping up unemployment, adverse exchange rate, rising factor costs, underutilization of installed plant capacity and rising prices of finished goods, among others. To stay this tide, researchers such as Sargeaunt (1985), Kurtz (1992), Lucey (2000) and Taha (2008) posit that the use of scientific methods, particularly linear programming, in the allocation of scarce resources is of vital importance to the manufacturing industries to boost output.

Business managers at Emene Industrial Layout have not adopted scientific quantitative methods as a major tool in decision making. The researcher conducted a survey of ten major manufacturing firms at the layout-Golden Plastic Industry Limited, Okema Plastics, Niger Gas, Robertson Aluminum, Florak Structural Aluminum, Sunrise Flour Mills, Niger Steel, Dunemo Fabrics, Emenite, and ANAMMCO- and discovered that non of them uses linear programming technique or other quantitative methods in production decision-making. They rather adopt traditional accounting methods such as cost-volume-profit analysis, standard-costing-and-variance analysis, budgeting and budgetary control, in trying to provide lasting solution to the challenge of optimal decision making. Some others simply use intuition or the trial-and-error method.

Golden Plastic Industry Limited, Emene produces eight different products. It has the necessary physical facilities to apply linear programming techniques in determining the quantity combination of the different types of products that maximize profit (optimal product mix). Computer facilities are adequate, yet the firm uses the trial-and-error method in determining its product mix. Some questions readily come to mind. First, what are the major reasons why manufacturing firms at Emene Industrial Layout do not use linear programming in major decision-making? Secondly, what in fact are the benefits of applying linear programming technique in determining the product mix of Golden Plastic Industry Limited, Emene? Can Golden Plastic Industry Limited (GPIL), Emene, make higher profit by employing skilled manpower to introduce the use of linear programming in determining its product-mix, considering the additional cost of doing so?

This paper will first highlight the peculiarities of applying linear programming technique by manufacturing firms at the Emene Industrial Layout, and show that such peculiar problems could be surmounted by employing skilled manpower. Secondly, it intends to establish that it would have been more profitable to use the linear programming technique in determining the product-mix of Golden Plastic Industry Limited despite the additional cost of acquiring additional manpower.

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[2] Emene Industrial Layout is one of the fastest growing industrial cluster in the eastern part of Nigeria.

[3] Enugu was the capital of Eastern Region and has given birth to at least five states in the present day Nigeria

## Objectives of the study

The main objectives of the study are:

1. To formulate a linear programming model that would suggest a viable product- mix to ensure optimum profit for GOLDEN PLASTIC Industries Limited, Emene.
2. To highlight the peculiarities of using linear programming technique at the Emene Industrial Layout and prove that despite the obstacles, the application of the technique in determining the product-mix of GOLDEN PLASTIC Industries Limited, Emene would be more profitable than otherwise.
3. To encourage manufacturing industries at the Emene Industrial Layout to adopt the application of linear programming technique in determining their product- mix. It seeks to illustrate the profitability of using the technique despite the peculiarities of applying it in the particular economic environment.

Bearing these objectives in mind, a post optimal analysis of the established profit maximization model would be attempted to help the management in adjusting its decisions in the face of increases or decreases in demand, resource prices and or availability of raw materials. Finally, from the findings of the study, suggestions of how linear programming method could be widely applied in business decision making in Nigeria would be offered.

## Brief Review of Literature (Theoretical and Empirical)

There are diverse opinions on the applicability of the linear programming technique to different management decision-making processes. These opinions developed over a long period of time following continuous improvement on the application of the technique in solving practical business problems. Most literature in economic development supports the view that linear programming is a practical tool of analysis in allocating scarce resources to their optimal use and is of vital importance to the economies of underdeveloped countries. Tracing the history of linear programming method, it is a mathematical device developed by the mathematician, George Dantzig, in 1947 for planning the diversified activities of the U.S. Air Force connected with the problem of supplies to the Force. Afterwards, Dantzig suggested this approach for solving business and industrial problems. He also developed a powerful mathematical tool known as “simplex method” to solve linear programming problems (Dantzig, 1993).

In an allocation problem, when there are a number of activities to be performed, alternative ways of doing them, and limited resources or facilities for performing each activity in the most effective way, the management is faced with the problem of how best to combine these activities and resources in an optimal manner so that the overall efficiency is maximized. According to Charles, Cooper and Henderson (1963), this is known as optimization problem, and can be approached using mathematical programming. They further refer to linear programming as a uni-objective constrained optimization technique. This is because, according to them, it seeks a single objective of either minimizing or maximizing unknown variables in a model. In line with this, Gupta and Hira (2009) argue that linear programming deals with linear optimization of a function of variables known as objective function subject to set of linear equations and /or inequalities known as constraints. The objective function may be profit, cost, production capacity or any other measure of effectiveness which is to be obtained in the best possible or optimal manner. The constraints may be imposed by different resources such as market demand, production process and equipment storage capacity, raw material available, and so on. They further posit that programming implies planning and by linearity is meant a mathematical expression in which the expressions among the variables are linear.

Dowing (1992) advocates that the Lagrangian method should be used for any optimization subject to a single inequality constraint, the Graphic approach for optimization subject to only two inequality constraints, and the linear programming model for optimization subject to many inequality constraints. Supporting this view, Dwivedi (2008) posits that linear programming is of great use in making business decision because it helps in measuring complex economic relations and thereby, provides an optimum solution to the problem of resource allocation. According to him, linear programming technique thus, bridges the gap between abstract economic theories and managerial decision-making. Furthermore, he stressed that any linear programming equation should have three specifications, namely: objective function specification, constraint equation specification, and non-negativity requirement. Corroborating this view, several authors (Dowling, 1992, Dwivedi, 2008, Koutsoyiannis, 1979, Henderson and Quandt, 2003, etc) have given the general specification of the linear programming model.

Sarggeaunt (1965) groups, what he calls, the principal types of application of linear programming models under three headings, namely: blending and mix determination problems, planning and scheduling problems and distribution cost problems. He also notes other types of applications: for instance to plant location decisions, to personnel allocation problems, and to the analysis of a multi-plant production system to determine whether or not certain plants should be shut down as a result of high cost of production. Emory and Niland (1968) have listed similar types of problems to which linear programming is applied. They include: the blending of gasoline stocks, the formulations of various chemical products, the mixing of fertilizer, the manufacture of cement and the charging of electric furnaces. On the other hand, Hillier and Lieberman (2001) assert that although allocating resources to activities is the most common type of application, linear programming has numerous other important applications as well. In fact any problem whose mathematical model fits the very general format for the linear programming model is a linear programming problem.

Turban and Meredith (1991) agreeing with Dwivedi (2008), states that linear programming is one of the best known tools to management science. Management science methods are composed of three components; the decision (uncontrolled) variables, the environment (uncontrolled) parameters and results (dependent) variables. The linear programming model, according to them, is composed of the same components but they assume different names; the decision variables which we seek to determine, the objective function which we aim to optimize and the constraints we need to satisfy. But Koutsoyiannis (1979) insists that linear programming can be considered as providing an operation method for dealing with economic relationship which involves discontinuities. She however maintains that neither economic theory nor linear programming say anything about the implementation of the optimal plan or solution. They simply derive the optimal solution in any particular problem.

Most researchers (Wagner, 2007, Lucey, 2002, etc) posit that linear programming is an operations research technique. They maintain that it is among the most commercially successful applications of operations research with Wagner (2007) insisting that there are considerable evidence that it ranks highest in economic impact. He stresses that operations research adopts the view that managers can follow a fairly systematic process for solving problems. Therefore, it is possible to use a scientific approach to managerial decision-making. In his words, operations research technique is a scientific approach to problem-solving for executive management. Other areas of usage include: airlines, agriculture, oil refining, education, energy planning, pollution control, transportation planning and schedule production planning, research and development, health care system and so on.

Emory and Niland (1968) listed the most difficult problems in using linear programming technique as: the recognition of the necessary restrictions (constraints or row equations) and the possible alternatives (the column vectors) and the expression of these in linear equations. The difficulty is to construct, in mathematical form, an accurate definition of the problem. Knowledge of the problem, the details of company operations, the mathematics of linear programming technique and ordinary ingenuity all come into play in the application of linear programming model. Computer storage capacity is also an important issue because sometimes, it is surprising how swiftly practical problems require matrices of a size which task even the larger modern computers. Experiences in the application of linear programming problems show that collection of necessary cost and technical data is both time-consuming and expensive. The linear programming problem, note Emory and Niland (1968), sometimes costs more than the actual solutions because of the costly nature of getting the relevant data needed. For Turban (1993), there is the problem of risks and uncertainties regarding the behaviour of customers, resources and commodity prices such that the management finds it difficult to choose the best decision among the alternative decision possibilities. He therefore recommends that managers must become more sophisticated, they must learn how to use new tools and techniques that are being developed in their field. These newly developed tools employ quantitative approach.

In pointing out limitations to its applicability, Kurtz (1992) notes that, often, very real organizational issues such as morale, consequences of strikes and internal power struggles are so difficult to quantify that they are ignored in the linear programming model used. Furthermore, considering the technicalities, managers who are conversant with its rigorous models resist its usage since they do not understand how it could be applied. But in his contributions, Taha (1992) lists other types of programming to include: integer, goal, quadratic, convex and stochastic. They differ, he maintains, in the kind of data they can handle and the kind of assumptions that are made. He finally illustrates how linear programming was applied to determine the feed mix to maintain a balanced ratio that includes calcium, protein, and fibre in the right proportion by the Ozark Poultry Farm.

Adam et al (1993) observe that optimization model aims at finding the best solution from a large or an infinite number of alternatives using a step-by-step improvement approach. Management seeks to find the best solution and an optimization algorithm identifies the steps for doing so.

Adam et al (1993) also provide an empirical evidence of the application of the linear programming model with the Multi-Band Enterprises. Finding the best mix of citizens band radio (CBs) and portable radio (PRs) to produce is a typical blending and mix determination problem. The Multi-Band Enterprises produces two products, PR and CB radios. Considering the fact that they can sell all that can be produced in the near future, there arose the problem of what quantity to be produced of the two types of radios per a month. The analysis of the operations department shows that each of the products require different amounts of sub-assembly, assembly and inspection labour. The labour availability for the month includes 316 hours of sub-assembly labour, assembly labour 354 hours and 65 hours of inspection labour. However, to produce each unit of CB requires 4 hours of sub-assembly labour, 5 hours of assembly and 0.5 hours of inspection labour, while each PR produced will attract 5 hours of sub-assembly labour, 3 hours of assembly labour and 0.10 hours of inspection labour. With all these information given above, the question then becomes; what quantities of these various bands of radio (CBs and PRs) the company will produce per a month bearing in mind that each CD sold contributes $50 to profit while each PR sold contributes $40 towards profit, in order to maximize profit. After the application of linear programming to this problem, the solution values show that the Multi-Band Enterprises will produce 632.31 of CBs, 126.15 PRs and this will give the maximum profit of $36,661.06. On the resources side, some hours of sub-assembly are unused while the assembly and inspection hours are extensively utilized.

Murugan and Manivel (2009) tried to demonstrate the use of linear programming technique by using linear interactive optimizer (LINDO) software to maximize the profit of Khadi and Village Industries Commission (KVIC) affiliated to Servodaya Sangham, run by a non-governmental organization (NGO). Servodaya Sangham engages in the production of three textile products, namely; cotton Khadi, Khadi ready-made and silk Khadi. Account records indicate that total profit earned by Servodava and Sangham was INR.22.31 million for the year. To produce each unit of the products requires the unit of raw materials cost of INR.32.74 million for cotton Khadi, INR.24.64 million for Khadi ready-made and INR.36.9 million for silk khadi. The unit labour cost of INR.7.36 million, INR.0.68 million and INR.7.58 million for cotton Khadi, Khadi ready-made and silk Khadi respectively is required while the unit overhead cost of INR.5.79 million, INR.0.49 million and INR.6.78 million for cotton Khadi, Khadi ready-made and silk Khadi respectively is incurred. The input availability of INR.94.28million, INR.15.62 million and INR.13.06 million for raw material, labour hour time and labour overhead time are used bearing in mind that each product will make a contribution of INR.9.23, INR.0.23 and INR.12.85 in million for cotton Khadi, Khadi ready-made and silk Khadi respectively. The application of linear programming suggests that the existing production volume would help to maximize the profit of Sanghan products to the tune of INR.24.752 million, noting that producing silk Khadi alone is economically profitable. The profit is INR.2.442 million higher than that of the present profit of INR.22.31 million.

Adeyemo and Otiero (2009) also tried to demonstrate that the linear programming model can be extended beyond the realms of Management Sciences and organizational decision departments to other areas such as Physical and Environmental Sciences. They used the application of Differential Evolution (DE) and Linear Programming (LP) to maximize total income (in South African Rand ZAR) of 2500 ha planting area where 16 crops are planted and constrained by water availability (using only 10mm$^3$ of irrigation water). It is found that a total income of ZAR 46,060,200 can be derived using linear programming. Ten strategies of DE are tested with this problem varying the population size (NP), crossover constant (CR) and weighing factor (F). It is found that strategy 1, DE/rand-1-bin, with values of NP, CR and F of 160, 0.95 and 0.5 respectively obtains the best solution most efficiently.

Kareem and Aderoba (2008) tried to show the effectiveness of adopting the linear programming model in maintenance and manpower planning using data from a cocoa processing industry in Akure, Ondo State of Nigeria. The result shows that only four maintenance crew out of the 19 employees are needed in that section to effectively carry out maintenance jobs in the industry. But in their own contributions, Nedim et al (2002) tried to demonstrate that risk analysis is necessary in order to maximize resources allocation efficiency and minimize the effects of risk environment. They used data from a sample of a company's products taking risk into account as the objective function. The result suggests that producing 5 units of $X_1$ generates 36% loss possibility. If decision makers aim risk not to exceed certain limits, then, variances should be used as constraints. The model suggests that producing 3 units of $X_1$ will decrease the objective function from $432 to $287.

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In another research, Susilawati et al (2002) extended the application of linear programming to dormitory development at Petra Christian University to calculate the number of rooms and area of each facility possible considering the limited space of 4,994.83 square meters and a maximum cash flow of Rp 392,952.00 which is discounted by 17% per annum for seven years. They were able to calculate the number and area of facilities such as bathrooms, dinning rooms, common rooms, cafeterias, bookshops, mini- markets, phone booths, sport facilities and parking spaces using special computer software which can solve mixed integer and LP model called SOLVER. The result suggests that all supporting facilities except garden need to be supplied. But in another study, Andre et al (2003) used linear programming to predict nutrients that are potentially low in a child's diet during the complementary feeding period such that the information gathered can be used to direct nutrition intervention programme initiatives because it suggests that foods rich in (foods fortified with) these limiting or problem nutrients should be introduced into local diets.

Several other empirical studies have shown the applicability of linear programming technique to diverse optimization problems. For instance, Koji and Gunner (2002) reveal the power of linear programming in image reconstruction by identifying and updating noisy pixels and applying the v-trick to directly specify the fraction of pixels to be reconstructed. But Dhanalakshm et al (2009) used linear programming in parts of speech tagging of Tarmil language in India by projecting an SVM methodology based on linear programming for implementing automatic Tamil POS tagger. In the study, "Quantitative Analysis of mineral mixtures using linear programming", Gerard (1986) demonstrated the determination of a multiphase system by applying X-ray Powder diffraction (XRO) and optimal point-counting methods as the means of obtaining quantitative information on the phase composition of a mixture. Cui et al (2006) also developed an efficient linear programming detector (LPD) for multiple-input multiple-output (MIMO) systems by employing multiple antennas at both the transmitter and the receiver of a high date-rate MIMO communication system to achieve remarkable high special efficiencies in rich scattering environments.

Based on both the theoretical and empirical literature provided above, there is the need to further explore and illustrate the application of linear programming model in business decision-making process in developing countries such as Nigeria. Determination of the product-mix of Golden Plastic Industries Limited located at Emene Industrial Layout, Enugu, South East of Nigeria, provides a good opportunity for such application. This paper will among other things try to establish the peculiarities of applying linear programming method in management decision-making with respect to the Nigerian situation and suggest useful ways of surmounting the problems.

## Methodology

Golden Plastic Industry Limited is chosen for this study for two main reasons. First, it uses the trial-and-error method in arriving at major management decisions even when the research department feels that a linear programming approach would have given a better result. Secondly, Golden Plastic Industry Limited produces eight different products which makes the determination of the quantity combinations of the products produced (product mix) an important and major management decision. The research is designed to cover one month, 1st to 31st May, 2011. Researchers have investigated the overall quantity combination of the eight products produced by Golden Plastic Industry Limited, Emene during the research period and the allocation of resources to the various products. This has been made possible by the records kept by the Production Line Manager and the Sales Department relating to the different brands of products produced by the firm, the technical coefficients, the raw materials available and their relative prices. Researchers also had personal interview with a representative of the management.

Researchers then applied linear programming to determine a new quantity combination. The total contribution to profit of each of the products for the month using the new quantity will now be compared with the total profit contribution made by the former product mix determined by the trial-and-error method. The problems encountered in the process will be noted and from personal interviews and relevant records, other peculiarities shall be established. When there are n choice variables and m constraints, the linear programming takes the general form with a linear objective function, a set of linear inequality constraints and a set of non-negativity restrictions as its major ingredients. The generalized n variable linear programme can be stated as below:

$$\begin{array}{lll} \text{Maximize} & \pi = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n & \text{(objective function)} \\ \text{Subject to} & a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n \leq r_1 & \\ & a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n \leq r_2 & \\ & \vdots & \\ & a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n \leq r_m & \end{array}$$

$x_j \geq 0$ ($j = 1,2,\ldots\ldots n$) (non-negativity restrictions) where $c_i$, $a_{ij}$ and $r_i$ are given constants. The variables $x_1, x_2, \ldots\ldots, x_n$ are called decision or structural variables. The problem is to find the values of the decision variables $(x_1, x_2, \ldots, x_n)$ which maximize the objective function $\pi$ subject to the m constraints and the non-negativity restriction on the $x_j$ variable. The resulting set of decision variables which maximize the objective function is called the optimal solution. This procedure is called "Simplex Algorithm).

**The Model**

The model for use in the present study is:

$$\begin{array}{ll} \text{MAXIMIZE } Z = & P_1X_1 + P_2X_2 + P_3X_3 + P_4X_4 \\ & P_5X_5 + P_6X_6 + P_7X_7 + P_8X_8 \end{array}$$

$$\begin{array}{lll} \text{SUBJECT TO:} & C_{11}X_1 + C_{12}X_2 + C_{13}X_3 + C_{14}X_4 & \\ & C_{15}X_5 + C_{16}X_6 + C_{17}X_7 + C_{18}X_8 & \leq B_1 \\ & C_{21}X_1 + C_{22}X_2 + C_{23}X_3 + C_{24}X_4 & \\ & C_{25}X_5 + C_{26}X_6 + C_{27}X_7 + C_{28}X_{28} & \leq B_2 \\ & C_{31}X_1 + C_{32}X_2 + C_{33}X_3 + C_{34}X_4 & \\ & C_{35}X_5 + C_{36}X_6 + C_{37}X_7 + C_{38}X_8 & \leq B_3 \\ & C_{41}X_1 + C_{42}X_2 + C_{43}X_3 + C_{44}X_4 & \\ & C_{45}X_5 + C_{46}X_6 + C_{47}X_7 + C_{48}X_8 & \leq B_4 \\ & C_{51}X_1 + C_{52}X_2 + C_{53}X_3 + C_{54}X_4 & \\ & C_{55}X_5 + C_{56}X_6 + C_{57}X_7 + C_{58}X_8 & \leq B_5 \\ & C_{61}X_1 + C_{62}X_2 + C_{63}X_3 + C_{54}X_4 & \\ & C_{65}X_5 + C_{66}X_6 + C_{67}X_7 + C_{68}X_8 & \leq B_6 \\ & C_{71}X_1 + C_{72}X_2 + C_{73}X_3 + C_{74}X_4 & \\ & C_{75}X_5 + C_{76}X_6 + C_{77}X_7 + C_{78}X_8 & \leq B_7 \\ & t_1x_1 + t_2x_2 + t_3x_3 + t_4x_4 & \\ & t_5x_5 + t_0x_0 + t_7x_7 + t_8x_8 & \leq T \\ & X_i \geq 0 \quad (\text{where } I = 1,2,\ldots,8) & \end{array}$$

where:

Z = total profit contribution of the various products of GPIL for the month of May, 2009.

$P_{1\ldots.8}$ = profit contribution coefficients i.e. the numerical values that express the per unit contribution to the profit equation.

$X_{1\ldots8}$ = the set of unknown we are seeking to determine i.e. the various products produce by GPIL.

C = technological coefficients i.e. the numerical values that express the per unit usage of the right hand side.

$B_{1\ldots.8}$ = the resource values that we seek to fully utilize.

T = he maximum labour time available for production within the production period (in hours).

$t_{1\ldots.8}$ = the labour time required to produce one unit of the various products of GPIL.

Estimates of the variables will be presented in tables. The optimum values of the different brands produced by the firm will show the combination (product mix) obtained through the application of linear programming model. The tables will also show those resources that are abundant and those that are in short-fall.

**The Firm and its Products:**

Golden Plastic Industry Limited (GPIL), Emene is engaged in the production of different sizes, shapes, and lengths of plastic pipes known as PVC pipes. These products are differentiated by their sizes, thickness and length. The pressure pipes used for circulating tap water is usually thicker than the waste pipes used in water system toilets and bathrooms. The products of GPIL include the following:

110mm by 5.4m thick pressure pipes
75mm by 5.4m light pressure pipes
63mm by 5.4m thick pressure pipes
50mm by 5.4m waste pipes
40mm by 5.4m thick pressure pipes
32mm by 5.4m thick pressure pipes
25mm by 5.4m conduit pipes

20mm by 5.4m thick pressure pipes.

In order to produce these pipes, the firm requires different materials in different combinations. It requires machines of different types and sizes, skilled and unskilled labour and raw materials. But for the purpose of this research work, we shall concentrate on raw materials and two factors: man and machine hours needed for production from the 1st day to the 31st day of May, 2011. Other factors are held constant. The major raw materials used by the firm in the production of the above products include:

- Resin (the major raw material)
- Calcium carbonate
- Tio 2
- Stabilizer
- Cast
- Carbon black
- Blend

The raw materials are mixed in different proportions and as such take different percentages of the production costs.

**Table 1**: Unit Profit Contribution by the various Products (see appendices). Based on all the information provided, Golden Plastic can be translated into the model above, thus:

$$\begin{aligned}\text{MAXIMIZE } Z = {} & 30X_1 + 40X_2 + 25X_3 + 15X_4 \\ & + 30X_5 + 25X_6 + 15X_7 + 35X_8\end{aligned}$$

$$\begin{aligned}\text{SUBJECT TO: } \quad & 516X_1 + 344X_2 + 322.50X_3 + 40X_4 \\ & + 279.50X_5 + 137.60X_6 + 14.40X_7 + 86X_6 && \leq 3{,}500{,}000 \\ & 180X_1 + 120X_2 + 112.50X_3 + 21X_4 \\ & + 97.50X_5 + 48X_6 + 6X_7 + 30X_8 && \leq 1{,}500{,}000 \\ & 60X_1 + 40X_2 + 37.50X_3 + 7X_4 \\ & + 32.50X_5 + 16X_6 + 2X_7 + 10X_8 && \leq 300{,}000 \\ & 108X_1 + 56X_2 + 67.50X_3 + 5X_4 \\ & + 58.50X_5 + 28.80X_6 + 3.20X_7 + 18X_8 && \leq 500{,}000 \\ & 48X_1 + 72X_2 + 30X_3 + 4X_4 \\ & + 26X_5 + 12.80X_6 + 1.20X_7 + 8X_8 && \leq 350{,}000 \\ & 84X_1 + 16X_2 + 52.50X_3 + 6X_4 \\ & + 45.50X_5 + 22.40X_6 + 2.80X_7 + 14X_8 && \leq 700{,}000 \\ & 204X_1 + 152X_2 + 127.50X_3 + 17X_4 \\ & + 110.50X_5 + 54.40X_6 + 10.40X_7 + 34X_8 && \leq 2{,}600{,}000 \\ & 0.0213X_1 + 0.0179X_2 + 0.0180X_3 + 0.0143X_4 \\ & + 0.0086X_5 + 0.0069X_6 + 0.0054X_7 + 0.0043X_4 && \leq 648\end{aligned}$$

**Data Analyses and Presentation of Findings**

The various estimated values of the optimization model for Golden Plastic Industry Limited are presented in 6 different tables (tables 2 to 7). Tables 2 and 3 present the primal and dual values of the estimates respectively while tables 4 to7 present the computer print-out of the post optimality values of the optimization model.

From table 3, the optimal solution of the problem is zero production for 6 out of the 8 products of GPIL. The production level of 25mm by 5.4m conduit pipes and 20mm by 5.4m thick pressure pipes yielded 114,317.2 and 7,136.564 respectively while the objective function yielded N1,964,532. But Golden Plastic could only make a total profit of N982,656 from 1st to 31st May, 2009. This obvious reduction in profit is partly attributed to random selection of product mix and resource allocation. Application of linear programming would have indicated to management that the company should produce only 114,317.2 of 25mm by 5.4m conduit pipes and 7136.564 of 20mm by 5.4m thick pressure pipes in order to make the maximum profit of N1,964,537. The other products should not be produced since their production adds more to cost than to profit.

The dual solution in table 3 re-affirms the authenticity of the optimal value of the objective function in the primal result. It also shows that while tio 2 ($Y_3$) and labour time ($Y_8$) are abundant due to their positive values, resin ($Y_1$), calcium carbonate ($Y_2$), stabilizer ($Y_4$), cast ($Y_5$), carbon black ($Y_6$) and blend ($Y_7$) are scarce due to their zero values. This means that they were consumed completely by the activities of the model. Therefore for the optimal solution to be improved, the scarce resources should be increased since any increase of the abundant resources will only make them more abundant without affecting the optimum solution.

The optimal values of the products; 25mm by 5.4m conduit pipes and 20mm by 5.4m thick pressure pipes increased and decreased with the post optimality C – when the resource budget and working time are increased and decreased respectively. However, it is important to note that the increase in the resources budget and working time in post optimality C yields a smaller optimal profit compared to the maximum profit of 1,964,537 naira in table 3 which suggests that there is a limit to which resources can be increased in other to achieve the objective function.

**Policy Implications of the Findings**

Based on the findings of the study, some policies that would help management in achieving the overall objective are highlighted.

i. The management of Golden Plastic Industry Limited, Emene, should ensure that the resources available to them are used in such a manner as to maintain the optimal values of the model in order to maximize profit.
ii. In the event of the management's desire to improve the established maximum profit within the limits of the model, only the funds allocated to the scarce resources should be increased by decreasing those of the abundant resources within the range.

## CONCLUSION

The study has successfully determined the product mix of Golden Plastic Industry Limited, Emene. In the process, the optimal quantities of the various PVC pipes to be produced within the study period in order to maximize profit were established. Also the status of the resources and the unit worth of each resource to the objective function were known. This is the advantage of going beyond mere knowledge of existing decision making tools to actual practical proof of its workability.

Another issue becomes how the managerial cadre of the productive firms at Emene Industrial Layout could be exposed to the rigorous steps involved in arriving at the optimal values of the linear programming model. From the researchers' personal observations in the course of this study, Golden Plastic Industry Limited, Emene has relatively few persons skilled in the Operations Research techniques who also possess a broad understanding of business environment and knowledge of the managerial roles and functions. As such, the firm should rely on outside consultants to bring this and other techniques to bear on management's decision problems. This can go a long way to assisting the management, at least, in the short run.

However, in the long run, many companies would, probably, gain from having permanent employees who can suggest opportunities to utilize these new techniques. They should be people who can effectively and efficiently interpret the results of mathematical analysis to top managers in the company's particular context, as well as possess the necessary competence in the utilization of computers for easy handling of the complex mathematical techniques involved.

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## APPENDICES

**Table 1:** Unit profit contribution by the various products

| | Production Cost (Naira) Per Unit | Unit Selling Price (Naira) | Unit Profit (Naira) |
|---|---|---|---|
| 1. 110mm by 5.4m thick pressure pipe | 1200 | 1230 | 30 |
| 2. 75mm by 5.4m light pressure pipe | 800 | 840 | 40 |
| 3. 63mm by 5.4m thick pressure pipe | 750 | 775 | 25 |
| 4. 50mm by 5.4m waste pipe | 100 | 115 | 15 |
| 5. 40mm by 5.4m thick pressure pipe | 650 | 680 | 30 |
| 6. 32mm by 5.4m thick pressure pipe | 320 | 345 | 25 |
| 7. 25mm by 5.4m conduit pipe | 40 | 55 | 15 |
| 8. 20mm by 5.4m thick pressure pipe | 200 | 235 | 35 |

**Table 2:** The primal optimal values of the decision variables

| Decision Variables | | Optimum Values |
|---|---|---|
| 110mm by 5.4 m thick pressure pipes | $X_1$ | 0 |
| 75mm by 5.4m light pressure pipes | $X_2$ | 0 |
| 63mm by 5.4m thick pressure pipes | $X_3$ | 0 |
| 50mm by 5.4m waste pipes | $X_4$ | 0 |
| 40mm by 5.4m thick pressure pipes | $X_5$ | 0 |
| 32mm by 5.4m thick pressure pipes | $X_6$ | 0 |
| 25mm by 5.4m conduit pipes | $X_7$ | 114,317.2 |
| 20mm by 5.4m thick pressure pipes | $X_8$ | 7,136.564 |
| Maximum Profit | | 1,964,537 |

**Table 3:** The Dual Resources Values (shadow) Prices

| Resources | | Values (shadow) Prices |
|---|---|---|
| Resin | $Y_1$ | 0 |
| Calcium Carbonate | $Y_2$ | 0 |
| Tio 2 | $Y_3$ | 2,742,291 |
| Stabilizer | $Y_4$ | 0 |
| Cast | $Y_5$ | 0 |
| Carbon Black | $Y_6$ | 0 |
| Blend | $Y_7$ | 0 |
| Labour time (hours) | $Y_8$ | 1762.12 |
| Optimum Value W | | 1,964,537 |

**Table 4:** Post Optimality A (unit profit increased)

| Decision Variables | Optimal Values |
|---|---|
| $X_1$ | 0 |
| $X_2$ | 0 |
| $X_3$ | 0 |
| $X_4$ | 0 |
| $X_5$ | 0 |
| $X_6$ | 0 |
| $X_7$ | 114,317.2 |
| $X_8$ | 1736.564 |
| Optimum Profit | 2,946,806 |

**Table 5:** Post Optimality B (Unit Profit Decreased)

| Decision Variables | Optimal Values |
|---|---|
| $X_1$ | 0 |
| $X_2$ | 0 |
| $X_3$ | 0 |
| $X_4$ | 0 |
| $X_5$ | 0 |
| $X_6$ | 0 |
| $X_7$ | 114,317.2 |
| $X_8$ | 7136.564 |
| Optimum Profit | 982,268.8 |

**Table 6:** Post Optimum C (Resource Budget and Working Time Increased)

| Decision Variable | Optimal Values |
|---|---|
| $X_1$ | 0 |
| $X_2$ | 0 |
| $X_3$ | 0 |
| $X_4$ | 0 |
| $X_5$ | 0 |
| $X_6$ | 0 |
| $X_7$ | 171,475.8 |
| $X_8$ | 10,704.85 |
| Optimum Value | 1,473,403 |

**Table 7:** Post Optimality D (Resource Budget and Working Capital Decreased)

| Decision Variables | Optimal Values |
|---|---|
| $X_1$ | 0 |
| $X_2$ | 0 |
| $X_3$ | 0 |
| $X_4$ | 0 |
| $X_5$ | 0 |
| $X_6$ | 0 |
| $X_7$ | 57,158 |
| $X_8$ | 3,568.282 |
| Optimal Profit | 982,259.87 |

**Appendix 1:** Decision Variables
$X_1$ = The quantity of 110mm by 5.4m thick pressure pipes to be produced.
$X_2$ = The Quantity of 75mm by 5.4m light pressure pipes to be produced.
$X_3$ = The quantity of 63mm by 5.4m thick pressure pipes to be produced.
$X_4$ = The quantity of 50mm by 5.4m waste pipes to be produced.
$X_5$ = The quantity of 40mm by 5.4m thick pressure pipes to be produced.
$X_6$ = The quantity of 32mm by 5.4m thick pressure pipes to be produced.
$X_7$ = The quantity of 25mm by 5.4m conduit pipes to be produced.
$X_8$ = The quantity of 20mm by 5.4m thick pressure pipes to be produced.

**Appendix 2:** Profit Contribution Coefficients (Given Constants)

Profit contribution coefficients represent the numerical values that express the per unit contribution to the profit equation (Z).

$P_1$ = The average net contribution by one unit of 110mm by 5.4m thick pressure pipe.
$P_2$ = The average net contribution by one unit of 75mm by 5.4m light pressure pipe.
$P_3$ = The average net contribution by one unit of 63mm by 5.4m thick pressure pipe.
$P_4$ = The average net contribution by one unit of 50mm by 5.4m waste pipe.
$P_5$ = The average net contribution by one unit of 40mm by 5.4m thick pressure pipe.

$P_6$ = The average net contribution by one unit of 32mm by 5.4m thick pressure pipe.
$P_7$ = The average net contribution by one unit of 25mm by 5.4m conduit pipe.
$P_8$ = The average net contribution by one unit of 20mm by 5.4m thick pressure pipe.

**Appendix 3:** Technology Coefficients (given constants)

In the model, technological coefficients represent the numerical values that express the per unit usage of the various raw materials in the production of the various products. For instance;
$C_{11}$ = Cost of Resin in producing one unit of 110mm by 5.4m thick pressure pipe.
$C_{21}$ = Cost of Calcium Carbonate used in producing one unit of 110mm by 5.4m thick pressure pipe.
$C_{71}$ = Cost of Blend used in producing one unit of 110mm by 5.4m thick pressure pipe.
$C_{12}$ = Cost of Resin used in producing one unit of 75mm by 5.4m light pressure pipe etc.

**Appendix 4:** Labour Time

Labour time here represents the labour time required to produce one unit of the various products of FIL. For instance;
$t_1$ = The labour time (in hours) required to produce one unit of 110mm by 5.4m thick pressure pipe.
$t_2$ = The labour time (in hours) required to produce one unit of 75mm by 5.4m light pressure pipe.
T = The maximum labour time available for production within the production period etc.

**Appendix 5:** Right-hand Side values (given constants)

These represent the resource values that we seek to fully utilize. For instance;
$B_1$ = The amount of money available within the production period for the purchase of Resin.
$B_2$ = The amount of money available within the production period for the purchase of Calcium Carbonate.